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Articulation between students' and teacher's activity during sessions about line symmetry

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The research presented here builds on an experimental work ran at the end of primary school (9–10 y. o. children) about line symmetry. We intend on questioning the factors that drive the evolution of geometrical activity of students and analyze in that purpose the articulation between student's and teacher's activity. We try to highlight the fact that learning in geometry relies on both an adaptation process when confronted to a task (individual and adaptationist dimension) and a collective and social construction mediated by interactions between teacher and students and between students.

Keywords: Line symmetry, adaptation, social construction, language.

This paper focuses on the teaching and learning of line symmetry at the end of primary school (9–10 year old children). The corpus that we study was collected during 5th grade classroom sessions where situations were used that had been created by a group mixing teachers from primary and secondary school and a researcher (one of the authors of this paper). The group had elaborated situations trying to take into account difficulties related to the transition from primary to secondary school and then implemented them in the classes of the teachers belonging to the group (Chesnais & Munier, 2013). Some videos of these classroom sessions are used here as a corpus for a research which intends on questioning the articulation between students' and teacher's geometrical activities. This research coordinates two different ways to understand the learning and teaching of geometry. The first one tries to describe the geometrical activity of students interacting with a given task, in all its complexity. It studies the factors that drive the evolutions of this activity in a movement for learning (Mathé, 2012; Bulf, Mathé, & Mithalal, 2011; Barrier, Hache, & Mathé, 2013). The other one, following up with previous work about everyday teaching practices and line symmetry, (Chesnais, 2009; Chesnais & Mathé, 2013; Chesnais & Munier, 2013), tries to investigate more precisely how teaching practices influence students' activity and hence students' learning, and also to get a better understanding of what drives teaching practices.

We take as a premise that learning in geometry relies on both an adaptation process when confronted to a task (individual and adaptationist dimension) and a collective and social construction mediated by interactions between teacher and students and between students. Our goal is here to highlight the complementarity of two approaches to get a better understanding of how the two processes articulate. After clarifying the elements of knowledge at stake in the learning of the concept of line symmetry, we will present the task and the methodology we used to analyze the productions of a pair of students and the teacher's activity during two sessions. We finally present on the results of this analysis and conclude.

ABOUT LINE SYMMETRY

What is mainly aimed at in 5th grade in France about line symmetry is that pupils understand an "instrumental definition" of symmetrical figures: two figures are symmetrical to each other with respect to a line if they are superimposable by folding along this line. They are also supposed to be able to find lines of symmetry on simple figures and to know some properties such as the fact that the mirror image is flipped over compared to the initial figure, the two figures are equidistant (in a global way) from the line and have same shape and dimensions.

At this stage, symmetry is mainly handled as a transformation acting on surfaces (2D-elements), and considered as restriction to the plane of a rotation of 180° around an axis included in the plane. Properties are then considered in a global manner and closely related to perception. However, working on symmetry might imply back-and-forth movements from relations between surfaces (and a line) and relations between 2D, 1D or 0D elements of the figures. For example, the dimension conservation property can either be perceived globally or focusing on the length of segment lines. In a similar way, equidistance to the line may be understood in terms of surfaces or elements of surfaces or event points. In fact, the idea of distance from a figure to a line is difficult at this level. It may refer either to the distance between the figure and the line perceived globally (Grenier, 1988), the distance between elements of the figure and the line perceived globally, the fact that the midpoint of a segment joining a point to its image belongs to the line, the distance from points to the line, seen as the length of the segment joining the point and its orthogonal projection on the line or its projection in a given direction.

About instruments, students at this level essentially use folding or tracing paper to control the symmetry of figures or to construct mirror images, mainly working on surfaces. Students might sometimes also use a ruler – as in the corpus presented below – which implies another way of considering figures (which is also the one that is at stake when working on grid paper): working with 1D-objects (segment lines, sides of surfaces). Switching from one way to the other makes it necessary to coordinate a view of figures as surfaces on one hand and as a network of segment lines on the other hand (what Duval (2005) calls "dimensional deconstruction"). The relation between symmetry and a movement in 3D-space is also less obvious.

Moreover, one of the goals of the work about symmetry in primary school is to make pupils overcome some wrong conceptions they might have about line symmetry, which particularly appear when working on surfaces (Grenier, 1988). Particularly those related to vertical lines, making them act as if the mirror image of a horizontal (resp. vertical) segment line is also horizontal (resp. vertical); the conception of alignment (resulting from the conjunction of vertical or horizontal lines and vertical and horizontal figures: the image of a segment line perpendicular to the axis is then aligned with its image); confusion with translation (which is related to the flipping over property); conception of symmetry as a transformation moving figures from one half-plane onto the other one (essentially related to folding) (Chesnais, 2009).

THE IMPLEMENTED TASK

The task is the second one in a sequence about line symmetry in a 5th grade class. In the first task, taking place during a previous session, students were supposed to predict what would paint stains become when folding a paper along various axes and then to identify some properties of line symmetry (equidistance of the two figures from the axis, shape and size conservation, flipping over property). In the present task, pupils were asked to draw the mirror image of a geometric figure (in the shape of an "L") with respect to a given line in various configurations (cf. Figure 1). Didactic variables (orientation of the figure and orientation of the axis) were used to make students

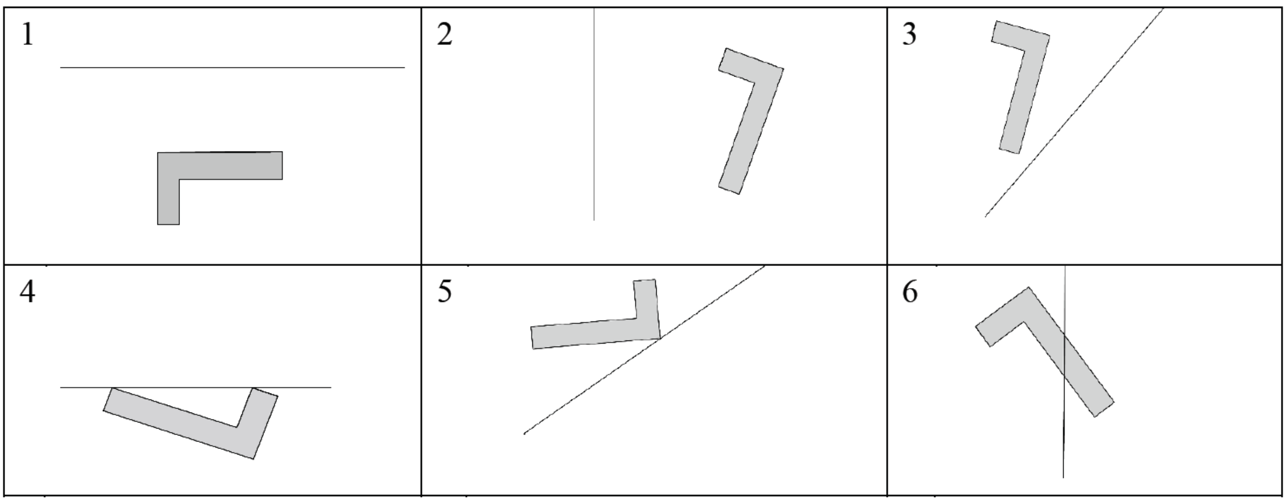

Figure 1: Configurations for the task

encounter one or the other of the properties and/or wrong conceptions.

Work was organized as follows: pairs of students were successively given six sheets – corresponding to each configuration. They were asked to draw the images approximately without folding they were allowed to use a pen and a ruler. A tracing paper on which the initial figure and the line were drawn for the first two cases and which was blank for the other four was provided to control their answer. The teacher went from one pair to another one to help them, and provided tracing paper when they were done drawing. A collective discussion with the whole class, based on some productions selected by the teacher, took place between the work in pairs on each case. The first three configurations were handled in a first session, which ended up by writing a couple of rules (mentioning the flipping over property, the equidistance to the line, conservation of shape and dimensions and the relation with folding), on a paper then posted on the wall. The work on the three other cases took place in a second session.

In this paper, we will particularly focus on one of the major properties of reflection: the "flipping over property" even if some other ones were also mobilized in the task. Mathematically speaking, it corresponds to the fact that line symmetry is an inversion. It is also related to the idea of "mirror image" or "reflection in a mirror". Materially speaking, it implies that a figure and its image are superimposable when the figure is flipped over, which means that a rotation of 180° degrees in 3D-space around a line included in the plane is applied to it. In practical terms, it can be obtained by folding the paper along the line or by flipping over tracing paper (and positioning it such that the line stays invariant). This property is difficult to express with common words because "flipping over" could also refer to a half-turn and then symmetry around a point whereas turning a page would end up flipping it over for example. Eventually, verbal language is not sufficient to handle this property and coordinating it with movements and material actions is necessary to make students identify and understand it (Chesnais & Mathé, 2013).

This property is part of what is at stake in the task. In the first case, flipping doesn't change the orientation of the sides of the figure (vertical stays vertical and horizontal stays horizontal). It does change it in all the other cases. Hence, it will be contradictory with what would other conceptions imply (especially the conceptions related to vertical and horizontal axis, and the one about alignment). In case 6, flipping can be hard to anticipate if the students have a conception of symmetry as transformation acting from one half-plane onto the other one (related to folding).

METHODOLOGY

We try to characterize students' activity, teacher's activity and the way they both articulate. Our indicators are mostly the material actions (folding, use of tracing paper and hands movements) and the interactions between students and students and teacher. We describe difficulties experienced by one pair of students throughout the work on the task, especially when the flipping over property is at stake. Based on the observation of this evolution, we then investigate how the task and social interactions participate in it towards an understanding of the flipping over property and symmetry. We also aim at understanding how the teacher identifies and uses students' activities and how she helps them completing the task.

GEOMETRICAL ACTIVITY OF A PAIR OF STUDENTS AND ITS EVOLUTION, BETWEEN ADAPTATIONISM AND SOCIAL PROCESS

In this part, we present an overview of what happened for each configuration during the sessions, focusing our attention on what the pair of students produced, the way they validated their answer and the content of the interactions with the teacher (when there are some) and of collective discussions. We aim at describing the evolution of geometrical activity of this pair of students throughout their work on the six configurations and try to identify what drives this evolution.

Configuration 1. They used a ruler to draw the image, taking into account the conservation of shape and dimensions, but without flipping the figure over (Figure 2). They probably didn't imagine the movement in 3D-space and then didn't anticipate the image. They used tracing paper for validation, but they couldn't figure out how to proceed differently when they found out that they did not get the right answer.

During the discussion with the class, the teacher chose to comment on their production. The class pointed out the absence of inversion. The teacher accompanied

Figure 3: Construction of configuration 2

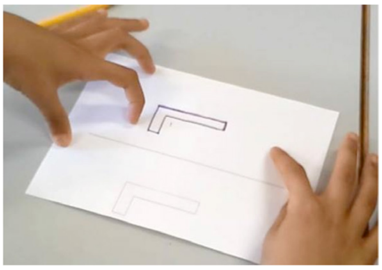

Figure 2: Production of configuration 1

the students trying to put this in words. Students used various wordings ("the other side", "in the other way around", "transferred", "turned over"). Their difficulties ended up in making the use of gestures necessary. The words "turned over" and gestures were also used by the teacher.

Configuration 2. One of the two students of the pair started by drawing globally the image with her finger: the position and orientation seemed correct and she flipped the figure over. Afterwards, she folded (not completely) the sheet of paper in order to find the precise position of one of the vertices of the image and then used a ruler to measure a first side of the figure to construct its image, transferring its length.

They finally gave then a third try, switching the orientation of the biggest part of the "L" but not the one of the smallest one (Figure 4).

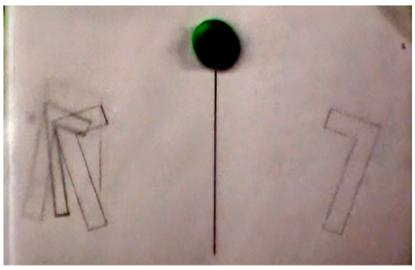

Figure 4: Final

The teacher chose again their production to comment during the final discussion. She makes the class explicit a mistake related to the conservation of shape (one of the students says "we had a 7, we get a 1"), but the link between the change of shape and flipping over the figure or elements of the figure is not pointed out. Flipping is not mentioned.

Configuration 3. They drew an image on the other side of the line, with the same size and same shape but without flipping it over (Figure 5). The students didn't seem to take into account the line and translated the figure, without imagining the rotation in 3D-space. Afterwards, they validated their answer using tracing paper (on which they draw both the figure and the line) but they slided the tracing paper, instead of flipping it over (Figure 6).

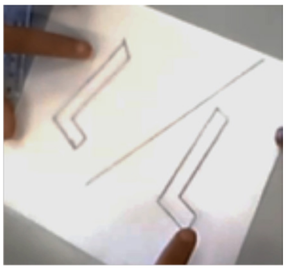

Figure 5: Production

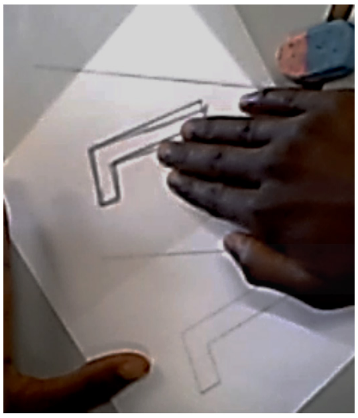

Figure 6: Validation

The teacher asked them how they controlled their answer and she pointed out the fact that the way they tried to control their answer was not correct, without mentioning a correct way to do it.

Part of the collective discussion was devoted to discussing this production. The class immediately pointed out the flipping over "problem". A work on putting it into words was conducted once again by the teacher and expressions like "in the same direction", "a little bit turned around", "completely turned around" were said.

"Turned over" was finally mentioned, repeated by the teacher and accompanied with gestures. At the end of this first session, the teacher elaborated a paper trail. She mentioned the "mistake often made" by this pair of students, pointing out that they often drew the figure "in the wrong direction". The teacher then wrote down that "the image has to be flipped over compared to the initial figure".

Configuration 4 (session 2). At first, the two students translated the figure along a direction given by the small part of the L (Figure 7); one of the two students, taking a wider look at it, realized the mistake. But constructing the image line by line made her do the same mistake again for her second try. Realizing it, they did a third try. The image was flipped over, but they did not switch the orientation, extending the sides of the small part of the L (Figure 7) and acting as if the line of symmetry was perpendicular to these sides. The other student tried to modify it so that the image touches the line at the same place as the initial figure (Figure 7). Taking into account the conservation of the shape led them to end up at the final production (Figure 7).

Trying to validate their answer, one of them drew the initial L and the line on tracing paper and then tried to turn the paper around (keeping it in the horizontal plane). The other one took it, flipped it over and replaced it so that the two lines matched.

The teacher then started a discussion with the pair. She placed the tracing paper sheet in its original position, then flipped it over and placed it such that the two lines matched. She tried here to link explicitly the flipping over movement of the tracing paper sheet and the flipping over property characterizing the relation between the initial L and its image (considered as relation between two figures in the plane). She then accompanied the two students to interpret feedback from superimposition of their production with the flipped over tracing paper. In particular, she showed them that the orientation of the bigger part of the L was correctly switched whereas the orientation of the smaller part didn't match. "Your line is straight, prolonging the other one, while it should have been tilted like that (she showed the side of the flipped over L)". She helped them taking into account the relation between global flipping of the figure and change in the orientation of 2D elements of the figure (the smaller part of the L and the bigger part), relation that was difficult for them to identify in the previous tasks.

The flipping over property was not mentioned during the collective discussion.

Observing the resolution of this four cases, we see the difficulties students encountered to perceive and take into account the fact that the initial figure and its image are flipped over, one compared to the other. They often anticipate approximately the correct position and orientation of the image in a global way, as well as they realize their mistakes when they look globally at their production after drawing. But drawing the image with a ruler makes them consider the figure in a very different way. It is not seen as a surface (2D) anymore, but as a grid of segment lines (1D) which images have to be constructed separately. Students seem then to forget about the symmetry as rotation of 180° degrees around the line in 3D-space and they keep working in the plane. Yet they don't have any other definition of symmetry than this instrumental definition (see above), which refers to folding (real or simulated). The different attempts for case 2 show that it is difficult for them to articulate the switch of orientation for each of the segments (or 2D elements of the figure – smaller or bigger part of the L) and conservation of shape. Students also have difficulties using tracing paper to control their productions,

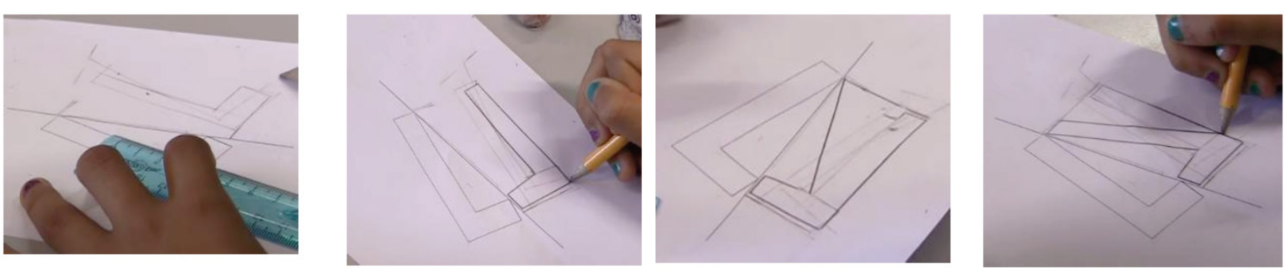

Figure 7: Try 1 | Try 3 | Try 4 | Final production

which makes it hard to use pragmatic feedback as we could observe about cases 2 and 3.

Configuration 5. At first, one of the two students drew an image, flipped over compared to the initial figure, but with an approximate orientation. The other one changed the drawing to get a better orientation and a better corresponding size. (Figure 8)

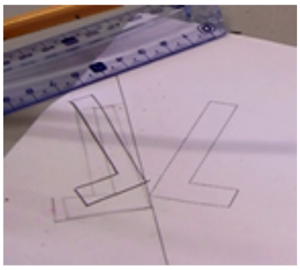

Figure 8

The collective discussion dealt with the question of the difference between "turned around" and "flipped over", about the productions of students who applied a symmetry around a point. Flipping over (rotation in 3D-space of 180° around the line) was again pointed out by the teacher, as opposed to rotation in the plane. Words were accompanied by a lot of gestures.

The teacher then pointed out the link between the need for flipping over and folding, which had been used to construct mirror images during the previous session. She folded a sheet of tracing paper so that folding makes the initial L flip over.

Configuration 6 The pair of students produced a flipped over figure, with same size and same shape as the initial L (Figure 9). The orientation of each part of the figure was correctly reversed. The invariance of points of the line was only partially completed.

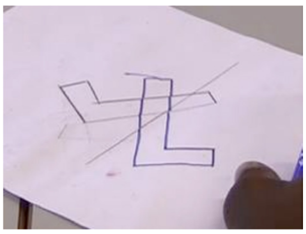

Figure 9

The teacher pointed it out during the collective discussion that followed.

The analysis of students' activity on the first four configurations showed that the interactions between the students and the task didn't seem sufficient to cause an evolution in their activity towards complete conceptualization of the flipping over property. However, an evolution has occurred in their ability to construct and validate, towards a better consideration of this property: their first idea was to translate the figure without flipping it over for the first four configurations, but they flipped it over directly for configurations 5 and 6. How can we understand what drove this evolution?

What we chose to observe leads us especially to pay attention to oral interactions during task completing. Consistently with our theoretical frame, this leads us to identify that, in this example, the evolution of pupils' activity is caused by a double process, adaptationist (students interacting with a task) and social, (essentially here through interactions between students and teacher). How do both interactions between the students and the task and interactions between students and teacher articulate to contribute to the evolution of the ability of students to acknowledge the flipping over property?

We distinguish two types of interactions: the interactions between the pair and the teacher, when she comes to talk to them during their validation; the interactions between the students and the teacher during collective discussions. Each of these types is a place where teacher and students' activities articulate, completing interactions between students' and the task, in order to make their understanding of the task and of the 'milieu' change. During the interactions between the students and the teacher, we identify three objectives of the teacher: helping them to identify mistakes (about the alignment of segments, in configuration 4), asking them to use tracing paper to control their answer to make them go back to material actions (configuration 3), and helping them using tracing paper correctly to flip it over and replace it so that the two lines match: what is at stake here is the link between the flipping over of the figure and the switch in the orientation of 1D elements. The teacher accompanies them in what interactions with the task was not sufficient to ensure. We suggest that, among these interventions, some have a productive function (Robert, 2008) (help students to complete the task) and some have a more constructive function (Ibid.) (help students transform activity into knowledge).

During collective discussions, the teacher based the debate on students' productions and mistakes. She made students explicit the flipping over property, then put it into words, and decontextualized it. However, she didn't emphasize the link between this property and the material action of folding or flipping tracing paper as much as she does when interacting with the two students (except for configuration 6).

RESULTS AND CONCLUSION

Our analysis of the evolution of productions of a pair of students on a task about symmetry allowed us to identify factors of this evolution: the feedbacks provided by the task but also interactions between students and between students and teacher. It points out how an adaptationist and a social process intertwine to ensure this evolution.

Finally, we claim that, in the observed sessions, students' progression towards a better consideration of the flipping over property results from articulation of four types of interactions between students and milieu:

- interactions between the students and the task, their use of instruments and the way they adapt to pragmatic feedback coming from the control with tracing paper;
- the verbal interactions between the two students in the pair;
- the interactions between the pair and the teacher, when she comes to talk to them during their validation;
- the interactions between the students and the teacher during collective discussions.

This study also informs on the way the teacher's activity may articulate with students' one: choice of the task, interactions with them during research, use of productions during collective discussions. Finally, it points out that linking various dimensions of activity (material, verbal and conceptual) is a condition to ensure learning.

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